



UNIVERSITY OF
SOUTH CAROLINA

What Do Analysts Do?

Duncan Wright
AWM Workshop
March 2, 2018

► Calculus



- ▶ Calculus
- ▶ Complex Analysis



- ▶ Calculus
- ▶ Complex Analysis
- ▶ Analysis on Function Spaces



- ▶ Calculus
- ▶ Complex Analysis
- ▶ Analysis on Function Spaces
- ▶ Mathematical Physics



Calculus

- ▶ Limits



Calculus

- ▶ Limits
- ▶ Continuity



Calculus

- ▶ Limits
- ▶ Continuity
- ▶ Derivatives



Calculus

- ▶ Limits
- ▶ Continuity
- ▶ Derivatives
- ▶ Series



Complex Analysis

▶ $f(x + iy) = u(x, y) + iv(x, y)$



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- ▶ Differentiable on an open set implies infinitely differential on that open set!



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- ▶ Linear operators on the linear operators on \mathbb{C}^n are just $n^2 \times n^2$ complex matrices.



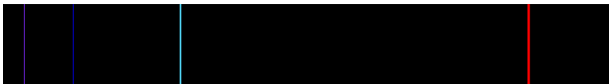
Mathematics and Physics

“A good physicist uses formalism as a poet uses language. He justifies the neglect of the commands of rigor by an eventual appeal to physical truth, as a mathematician cannot permit himself to do.”

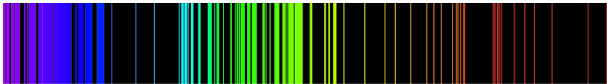
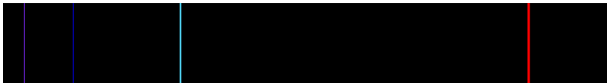
-Yu. I. Manin



Spectrum of Elements



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Mathematical Formulation of Quantum Mechanics

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Finite dimensional Hilbert spaces are just \mathbb{C}^n !!!



Questions?

